Chapter 24 Electric Potential

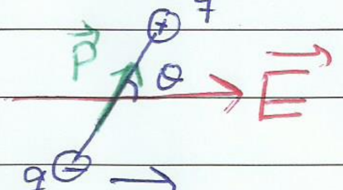
Conservative Forces & Potential Energy:

$$\vec{F}_g = m\vec{g} \Longrightarrow U_g = mgy \text{ Joul}$$

$$F_{spring} = -kx \Longrightarrow U_{spring} = \frac{1}{2}kx^2 \text{ Joul}$$

Electric Force acting on electric dipole

$\vec{E}$ on $\vec{P} = q\vec{d}$



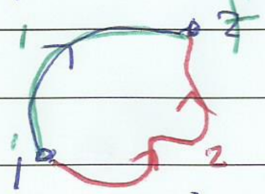
$$\Longrightarrow U = -\vec{p}\cdot\vec{E}$$
$$= -pE\cos\theta \text{ Joul}$$

Properties of conservative Forces:-

1) Work done Against Conservative Force is stored as energy, Called Potential energy.

2) Work done by Conservative force is path independent.
Work done by conservative force depends on initial point & final point

$$\left(W^{cons.}_{i\to f}\right)_1 = \left(W^{cons.}_{i\to f}\right)_2$$



3) Work done by Conservative force $_{i\to f}$ $= -\Delta U$
$$= -[U_f - U_i]$$

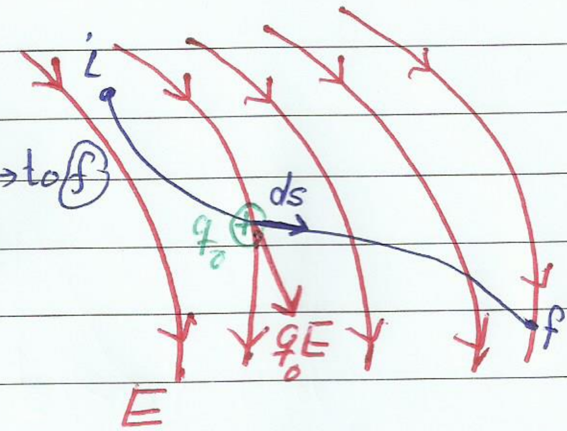
U is the Potential Energy depends on the State.

4) Mechanical Energy is Conserved Under the influence of conservative Force

$$(K+U)_i = (K+U)_f \quad , \quad K = \frac{1}{2}mv^2 \text{ Joul}$$

Electric Potential Energy U:

E is a nonuniform Electric Field



q_0 is a test charge moves from i $\rightarrow$ to f along the shown path

$$dW_E = q_0 \vec{E} \cdot \vec{ds}$$

$$(W_E)_{i \to f} = \int_i^f q_0 \vec{E} \cdot \vec{ds} \quad \text{Joul}$$

But: The electric Force $\vec{F}_E = q_0 \vec{E}$ is Conservative Force

$$W_{E\, i \to f} = -\Delta U = -[U_f - U_i]$$

$$U_f - U_i = -W_{E\, i \to f} = -\int_i^f q_0 \vec{E} \cdot \vec{ds}$$

$$U_f - U_i = (-) q_0 \int_i^f \vec{E} \cdot \vec{ds} \quad \text{Joul} \quad (1)$$

the Zero level for Electric Potential Energy is when q_0 is at ∞

$$U_f - U_\infty = (-) q_0 \int_\infty^f \vec{E} \cdot \vec{ds} \quad , \text{ let } U_\infty = 0$$

$$U_f = (-) q_0 \int_\infty^f \vec{E} \cdot \vec{ds} \quad (2)$$

Electric Potential energy For q_0 in an Electric Field.

U_f = (-) Work done by $\vec{E}$ in moving q_0 ($\infty \rightarrow f$)

Electric Potential V:

$$U_f - U_i = (-)q_0 \int_i^f \vec{E} \cdot \vec{ds} \quad J$$

$$\frac{U_f}{q_0} - \frac{U_i}{q_0} = -\int_i^f \vec{E} \cdot \vec{ds} \quad J/C$$

$$\boxed{V_f - V_i = -\int_i^f \vec{E} \cdot \vec{ds}} \quad (1) \quad J/C \equiv Volt$$

to find V at a certain point let $V_\infty = 0$

$$V_f - V_\infty = -\int_\infty^f \vec{E} \cdot \vec{ds}$$

$$\boxed{V_f = -\int_\infty^f \vec{E} \cdot \vec{ds}} \quad (2) \quad J/C \equiv Volt$$

V at a certain point = (−) Work done by $\vec{E}$ in moving +1C from $\infty \rightarrow$ to the Point

$$U = qV$$

Joul, C, Volt

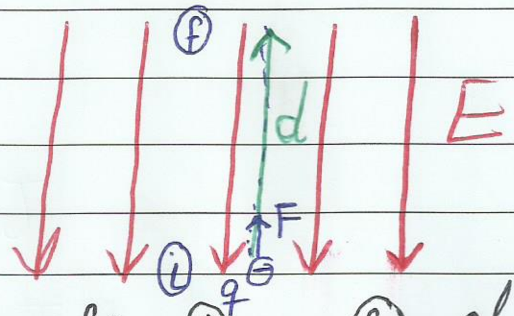
In this Course We will find V from E

1) We will find V due to a point charge

2) We will find V due to a Set of Point charges

3) We will find V due to a a Continous Charge distribution

Sample Problem 24.01

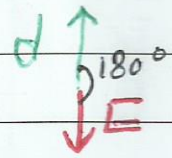


$E = 150\ N/C$ downward

$q = -e$
$\quad = -1.6\times10^{-19}\,C$

due to $\vec{E}$, the election moves from (i) $\longrightarrow$ (f) $d = 520m$ upward

1)
$$W^{E}_{i\to f} = \int_i^f q\vec{E}\cdot\vec{ds} = q\vec{E}\cdot\vec{d}$$



$$W^{E}_{i\to f} = (-1.6\times10^{-19})(150)(520)\cos180^\circ$$
$$= 1.2\times10^{-14}\,J$$

2) find $\Delta U = U_f - U_i$?

$$\Delta U = U_f - U_i = -W^{E}_{i\to f} = -1.2\times10^{-14}\,J$$

3) find $\Delta V = V_f - V_i$?

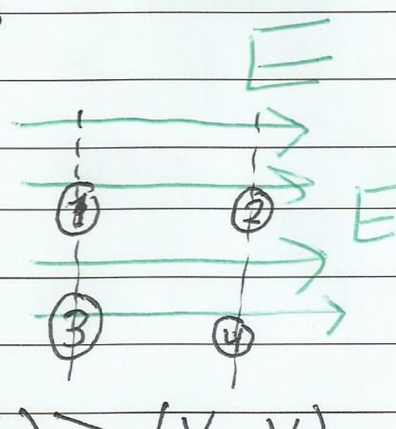
$$\Delta U = q\,\Delta V$$
$$U_f - U_i = q(V_f - V_i)$$
$$V_f - V_i = \frac{U_f - U_i}{q} = \frac{-1.2\times10^{-14}}{-1.6\times10^{-19}} = +7.5\times10^{4}\,V$$

$V_f > V_i$

Note: When you move with $\vec{E}$, V will decrease

When you move opposit to $\vec{E}$

V will increase.



$\Longrightarrow V_1 > V_2$

$V_1 = V_3$,, $V_2 = V_4$

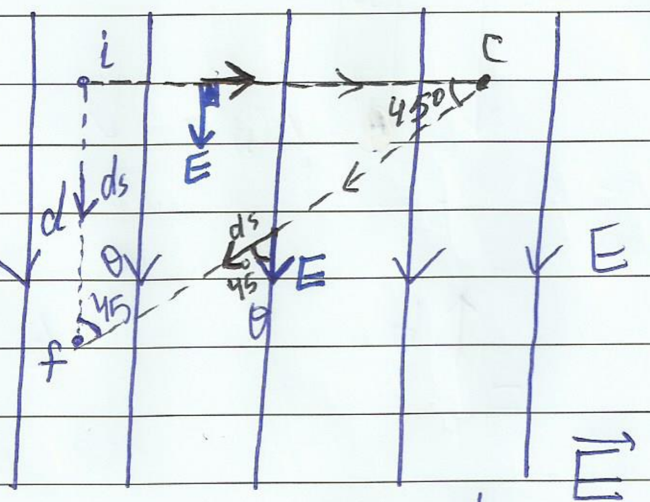
$(V_1 = V_3) > (V_2 = V_4)$

4

Sample Problem 24.02 ΔU is Path independent لا يعتمد
ΔV is Path independent على مسار

$\vec{E}$ is Uniform downward

$d_{if} = d$



a) find $V_f - V_i$ by moving +1C directly from $i \to f$

$$V_f - V_i = -\int_i^f \vec{E} \cdot \vec{ds}$$

$$= (-)\, \vec{E} \cdot \vec{d}_{i \to f}$$

$$= (-)\, E d \cos 0$$

$$V_f - V_i = -Ed \Rightarrow V_i > V_f$$

$\cos\theta = \dfrac{d}{d_{cf}}$

$d = d_{cf} \cos\theta$

b) find $V_f - V_i$ by moving (+) along the Path icf

$$V_f - V_i = -\int_i^c \vec{E} \cdot \vec{ds} - \int_c^f \vec{E} \cdot \vec{ds} \quad , \vec{E} = \text{Constant}$$

$$= -\vec{E} \cdot \vec{d}_{i \to c} - \vec{E} \cdot \vec{d}_{c \to f}$$

$$= -E d_{i \to c} \cos 90 - E d_{c \to f} \cos 45$$

$$= 0 - E d_{c \to f} \cos 45$$

from the graph

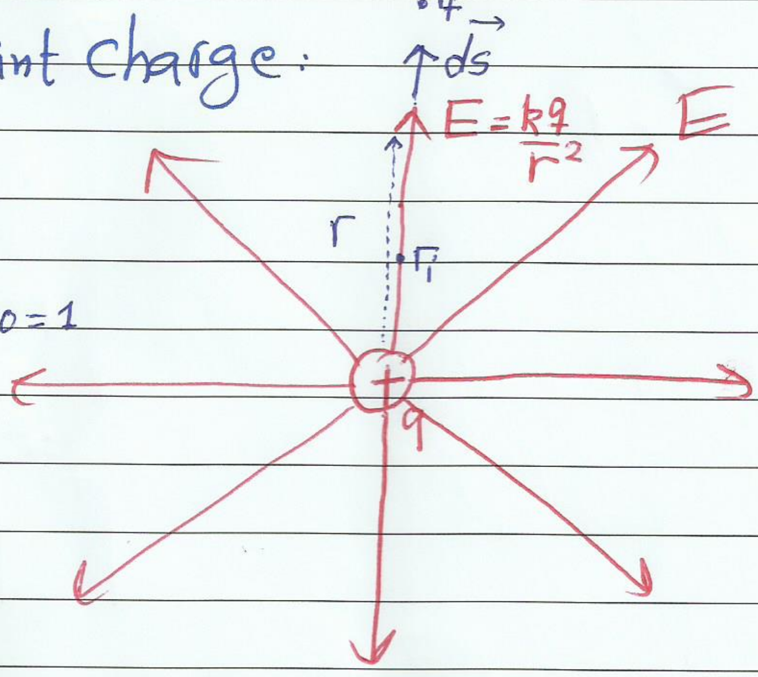
$\cos 45 = \dfrac{d}{d_{cf}}$

$d = d_{cf} \cos 45$

$$V_f - V_i = -Ed \Rightarrow V_i > V_f$$

as Part (a)

1) V due to a point charge:



$$E = k\frac{q}{r^2} = \frac{q}{4\pi\epsilon_0 r^2}$$

$$V_f - V_i = -\int_i^f \vec{E}\cdot\vec{ds} \quad , \cos 0 = 1$$

$$= (-)\int_{r_i}^{r_f} \frac{q}{4\pi\epsilon_0 r^2}\,dr$$

$$= \frac{-q}{4\pi\epsilon_0}\int_{r_i}^{r_f}\frac{dr}{r^2}$$

$$= \frac{-q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_i}^{r_f} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_i}^{r_f}$$

$$V_f - V_i = \frac{q}{4\pi\epsilon_0 r_f} - \frac{q}{4\pi\epsilon_0 r_i} \quad , \text{let } (i) \text{ at } \infty \;, V_\infty = 0$$

$$V_f - V_\infty = \frac{q}{4\pi\epsilon_0 r_f} - 0$$

$$\boxed{V_f = \frac{q}{4\pi\epsilon_0 r_f}}$$

for $+q$ $\quad V_f \to +$

for $-q$ $\quad V_f \longrightarrow -$

2) V due to a Set of point charges

$$V = V_1 + V_2 + V_3 + \cdots = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

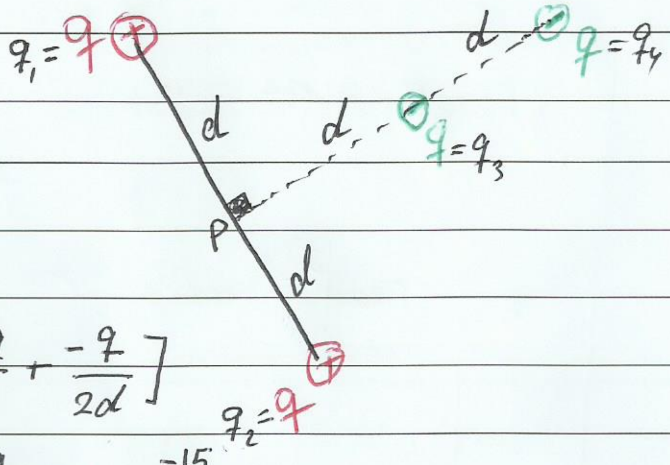
Solve Sample Problems 24.03 & 24.04

(24-31)

$q = 7.50\,fC$

$d = 1.60\text{ cm}$

find V_p ?

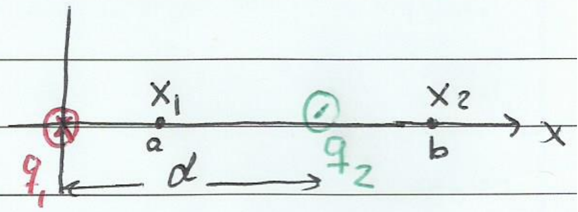


$$V_p = V_1 + V_2 + V_3 + V_4$$
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{+q}{d} + \frac{+q}{d} + \frac{-q}{d} + \frac{-q}{2d}\right]$$
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{+q}{2d}\right] = \frac{9\times10^{9}\times 7.5\times10^{-15}}{2(1.6\times10^{-2})}$$
$$V_p = \frac{4.22\times10^{-3}\text{ V}}{2} = \frac{4.22\text{ mV}}{2} = 2.11\text{ mV}$$

(24-7) $q_1 = +15e$

$q_2 = -5e$

$d = 2.40\text{ cm}$



find Values of x ? At which $V = 0$

a) At x_1 between them ?

$$V_a = \frac{+15e}{4\pi\epsilon_0 x_1} + \frac{-5e}{4\pi\epsilon_0 (d - x_1)} = 0$$

$$\frac{+15e}{4\pi\epsilon_0 x_1} + \frac{-5e}{4\pi\epsilon_0 (d - x_1)} = 0 \Rightarrow \frac{3}{x_1} = \frac{1}{d - x_1}$$

$$x_1 = 3d - 3x_1 \Rightarrow 4x_1 = 3d \Rightarrow x_1 = \frac{3}{4}d = {}^{+}18\text{ cm}$$

b) At x_2 ~~to the~~ the right of q_2

$$V_b = \frac{+15e}{4\pi\epsilon_0 x_2} + \frac{-5e}{4\pi\epsilon_0 (x_2 - d)} = 0 \Rightarrow \frac{15e}{4\pi\epsilon_0 x_2} = \frac{5e}{4\pi\epsilon_0 (x_2 - d)}$$

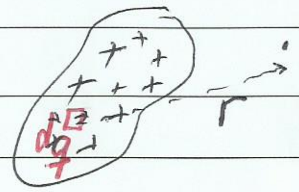
$$\frac{3}{x_2} = \frac{1}{x_2 - d} \Rightarrow 3x_2 - 3d = x_2$$
$$2x_2 = 3d \Rightarrow x_2 = 1.5d = 36\text{ cm}$$

Problem Repeat problem 7 for $\rightarrow q_1 = +5e$

$\rightarrow q_2 = -15e$

3) Potential due to a Continous Charge distribution:

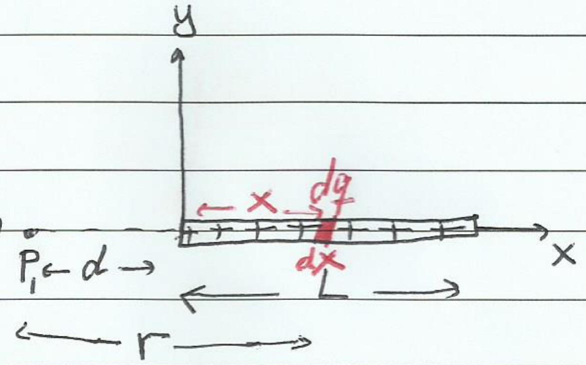
$$dV = \frac{dq}{4\pi\epsilon_0 r}$$



$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

I] V due to a Uniformly charged rod (Line of charge)

Problem(24-02)



thin Plastic rod
- → length = L = 12 cm
- → Q = +47.9 fC
- → $\lambda = \frac{Q}{L} = 400 fC/m$
$= +400 fC/m$

Find V at point P_1, a distance $d = 2.5 cm$ from the lef end of the rod?

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda dx}{4\pi\epsilon_0 (d+x)} \quad , \lambda = 4.00\times10^{-13} C/m \text{ Constant}$$

$$V_{P_1} = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(d+x)} = \frac{\lambda}{4\pi\epsilon_0} \ln(d+x)\Big]_0^L$$

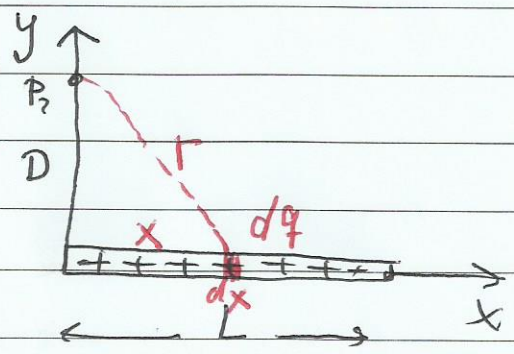
$$= \frac{\lambda}{4\pi\epsilon_0}\left[\ln(d+L) - \ln(d)\right]$$

$$V_{P_1} = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{d+L}{d}\right)$$

$$V_{P_1} = 9\times10^{9}\times4\times10^{-13}\ln\left(\frac{2.5+12}{2.5}\right)$$
$$= 36\times10^{-4}(1.75786)$$
$$= 6.33\times10^{-3} V$$
$$= 6.33 mV$$

Problem (24-6) Nonuniform

Linear charge density



The thin plastic rod: length = 12 cm, $\lambda = cx$

$c = 49.9\, pC/m^2$

a) Find V at p_2; on the y-axis at

$y = D = 3.56\,cm$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

$$V_{P_2} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda dx}{\sqrt{D^2+x^2}} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{(cx)dx}{\sqrt{D^2+x^2}}$$

$$= \frac{c}{4\pi\epsilon_0}\int_0^L \frac{x\,dx}{\sqrt{D^2+x^2}}, \quad \text{let } D^2+x^2 = u$$

$$du = 2x\,dx$$

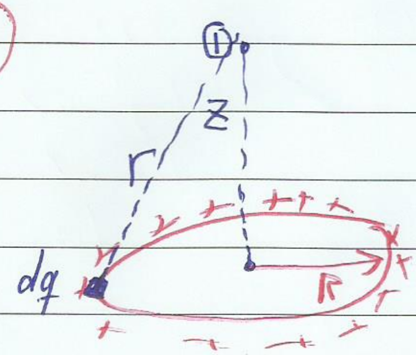
$$= \frac{c}{4\pi\epsilon_0}\int\frac{du/2}{u^{1/2}} = \frac{c}{8\pi\epsilon_0}\int u^{-1/2}du = \frac{u^{1/2}}{1/2}$$

$$= \frac{2c}{8\pi\epsilon_0}\left[\sqrt{D^2+x^2}\right]_0^L = \frac{c}{4\pi\epsilon_0}\left[\sqrt{D^2+L^2} - D\right]$$

$$V_{P_2} = \frac{c}{4\pi\epsilon_0}\left[\sqrt{D^2+L^2} - D\right] = 9\times10^9\left[49.9\times10^{-12}\right]\left[\sqrt{(0.0356)^2+(0.12)^2} - 0.0356\right]$$

$$= 0.04\,\text{Volt}.$$

V due to a Uniformly Charged Ring



Ring: radius = R, charge = +Q, $\lambda = \frac{Q}{2\pi R}$

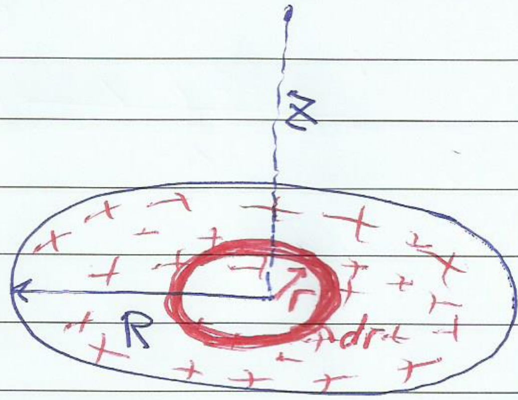
Find V at a point above the center a distance = z

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{dq}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

$$V_{ring} = \int\frac{dq}{4\pi\epsilon_0\sqrt{z^2+R^2}} = \frac{1}{4\pi\epsilon_0\sqrt{z^2+R^2}}\int dq = \frac{Q}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

V due to a Uniformly charged Disk

Disk → radius = R
→ charge = Q on one face
→ $\sigma = \frac{Q}{\pi R^2}$



Find V at a point above the center a distance (Z) from the center

Divide the disk to rings

each ring of → radius = r
→ width = dr
→ Area $dA = 2\pi r dr$
→ $dq = \sigma(2\pi r dr)$ ← charge on the ring
→ $dV = \frac{dq}{4\pi\epsilon_0\sqrt{r^2+z^2}}$ due to the ring

$$dV = \frac{\sigma(2\pi r dr)}{4\pi\epsilon_0\sqrt{r^2+z^2}} \quad \text{(ring Potential)}$$

$$V_{disk} = \int dV_{ring} = \frac{\sigma\pi}{4\pi\epsilon_0}\int_0^R \frac{2r dr}{\sqrt{r^2+z^2}}, \quad \text{let } u = r^2+z^2, \; du = 2r dr$$

$$= \frac{\sigma}{4\epsilon_0}\int \frac{du}{u^{1/2}} = \frac{\sigma}{4\epsilon_0}\int u^{-1/2} du = \frac{\sigma}{4\epsilon_0}\left[\frac{u^{1/2}}{1/2}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]$$

Calculating The Field E From the Potential V

$$dV = -\vec{E}\cdot d\vec{s} \Rightarrow -\frac{\partial V}{\partial s} = E \Rightarrow$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial y} \quad V/m$$

Sample Problem 24.05

$$V_{disk} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

Find E_z?

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\epsilon_0}\frac{d}{dz}\left[\sqrt{z^2+R^2} - z\right]$$

$$= -\frac{\sigma}{2\epsilon_0}\left[\frac{\cancel{1}\,2z}{\cancel{2}\sqrt{z^2+R^2}} - 1\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

Problem:

$$V_{ring} = \frac{Q}{4\pi\epsilon_0(R^2+z^2)^{1/2}}$$ Find E_z

$$E_z = -\frac{\partial V}{\partial z} = -\frac{Q}{4\pi\epsilon_0}\frac{\partial}{\partial z}(R^2+z^2)^{-1/2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{2}(R^2+z^2)^{-3/2}\cdot 2z\right]$$

$$E_z = \frac{Qz}{4\pi\epsilon_0(R^2+z^2)^{3/2}}$$ Ring

Problem (24-63) $V = 2x^2 - 3y^2$

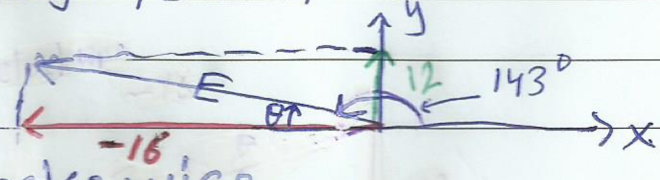
Find $\vec{E}$ at the point (4m, 2m)?

$$E_x = -\frac{\partial V}{\partial x} = -2(2x) = -4x$$

$$E_y = -\frac{\partial V}{\partial y} = -3(-2y) = +6y$$

$$\vec{E} = -4x\hat{i} + 6y\hat{j}$$

$$\vec{E} = -4(4)\hat{i} + 6(2)\hat{j} = -16\hat{i} + 12\hat{j} \text{ N/C or V/m}$$



$$\theta = \tan^{-1}\left(\frac{+12}{-16}\right) = -37^\circ$$

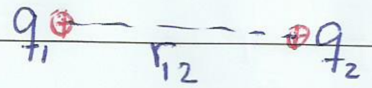
$\vec{E} = 20$ V/m, at 37° with $-x$ clocke wise

$= 20$ V/m, at 143° with $+x$

Counter Clock wise

Electric Potential Energy (U) of a System of Charged Parti

U between 2 point charges = Work must be done to put the 2 charges in their places by bringing them from ∞.



$$U_{12} = W_{q_1 \atop \infty\to} + W_{q_2 \atop \infty\to}$$

$$= 0 + q_2 V_1$$

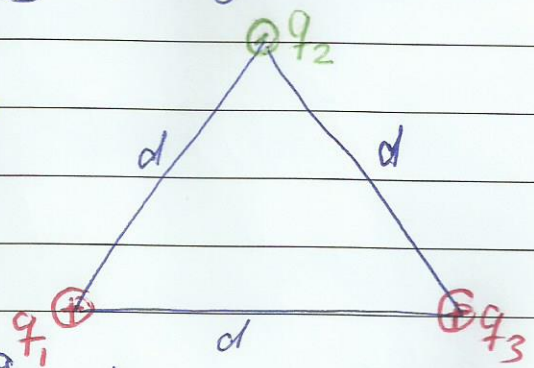
$$= 0 + q_2\left(\frac{q_1}{4\pi\epsilon_0 r_{12}}\right)$$

$$U_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \text{ Joul}$$

Sample Problem 24.06 Potential energy of a system of 3 charged Particles.

$q_1 = +q$, $q_2 = -4q$

$q_3 = +2q$, $q = 150\,nC$, $d = 12\,cm$

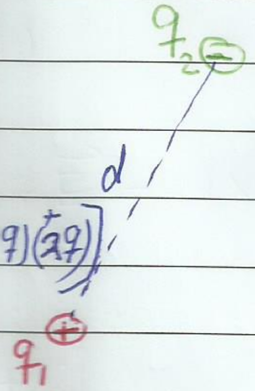


$$U = W_{q_1 \atop \infty\to} + W_{q_2 \atop \infty\to} + W_{q_3 \atop \infty\to}$$

$$= 0 + q_2\left(\frac{q_1}{4\pi\epsilon_0 d}\right) + q_3\left(\frac{q_1}{4\pi\epsilon_0 d} + \frac{q_2}{4\pi\epsilon_0 d}\right)$$

$$= \frac{q_1 q_2}{4\pi\epsilon_0 d} + \frac{q_1 q_3}{4\pi\epsilon_0 d} + \frac{q_2 q_3}{4\pi\epsilon_0 d}$$

$$U_{12} + U_{13} + U_{23}$$



$$= \frac{1}{4\pi\epsilon_0 d}\left[(+q)(-4q) + (+q)(+2q) + (-4q)(+2q)\right]$$

$$= \frac{1}{4\pi\epsilon_0 d} q^2 \left[-4 + 2 - 8\right]$$

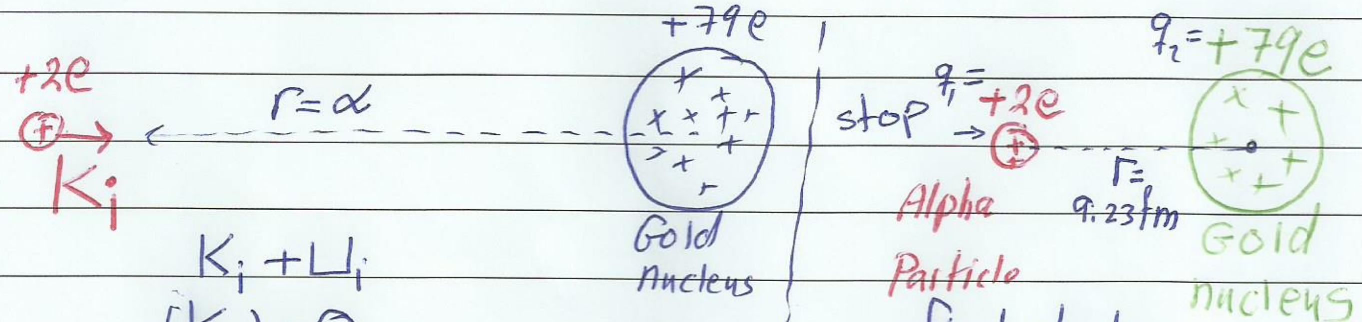
$$= \frac{-10 q^2}{4\pi\epsilon_0 d} = \frac{(-10) 9\times10^9 \times (150\times10^{-9})^2}{0.12} = -0.017\ V.C$$

$$= -0.017\ J$$

$$= (-)17\ mJ$$

Sample Problem: 24.07

Conservation of Mechanical energy With electic Potential energy:



final state

$$K_i + U_i = K_f + U_f$$

$$(K_i)_\alpha + 0 = 0^\alpha + \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$(K_i)_\alpha = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$= \frac{9\times10^{9} \times 2(1.6\times10^{-19})(+79)(1.6\times10^{-19})}{9.23\times10^{-15}}$$

$$(K_i)_\alpha = +3.94\times10^{-12}\,J$$

But $1eV = 1.6\times10^{-19}\,J$

$$(K_i)_\alpha = \frac{+3.94\times10^{-12}}{1.6\times10^{-19}} = 2.46\times10^{7}\ eV$$

$$= 24.6\ MeV$$

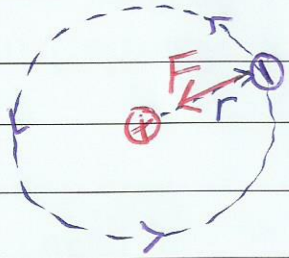
Units of Energy: Joul

$$kWh = (1000)(3600) = (1000\,\tfrac{J}{s})(3600\,s) = 3.6\times10^{6}\,J$$
$$kWh = 3.6\,MJ$$

$$eV = (1.6\times10^{-19}\,C)(1V) = 1.6\times10^{-19}\,J$$

Problem: Hydrogen Atom

In the H-atom, the electron moves in a Uniform circular motion around the nucleus, of radius $= 5.29 \times 10^{-11}$ m



$q_{electron} = -1.6 \times 10^{-19}$ C

$q_{nucleus} = +1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

1) Find the kinetic energy of the electron ?
2) Find the Potential energy of the electron ?
3) Find the Mechanical Energy of the electron ?

1) $F = \dfrac{q_e q_n}{4\pi\epsilon_0 r^2} = \dfrac{mv^2}{r}$ Coulomb's Law + Newton's 2nd Law

$$\frac{1}{2}\left[mv^2 = \frac{q_e q_n}{4\pi\epsilon_0 r}\right]$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q_e q_n}{4\pi\epsilon_0 r}\right) = \frac{1}{2}\left(\frac{(1.6\times10^{-19})^2}{5.29\times10^{-11}}(9\times10^{9})\right)$$

$$K.E = 2.18\times10^{-18}\,J = 13.6\,eV$$

2) $$U = q_e\left(\frac{q_n}{4\pi\epsilon_0 r}\right) = -1.6\times10^{-19}\left(\frac{9\times10^{9}\times1.6\times10^{-19}}{5.29\times10^{-11}}\right)$$

$$= (-)\,4.36\times10^{-18}\,J = (-)\,27.2\,eV$$

3) Mechanical Energy $= K.E + U = {}^{+}13.6 + {}^{-}27.2$

$$= -13.6\,eV$$

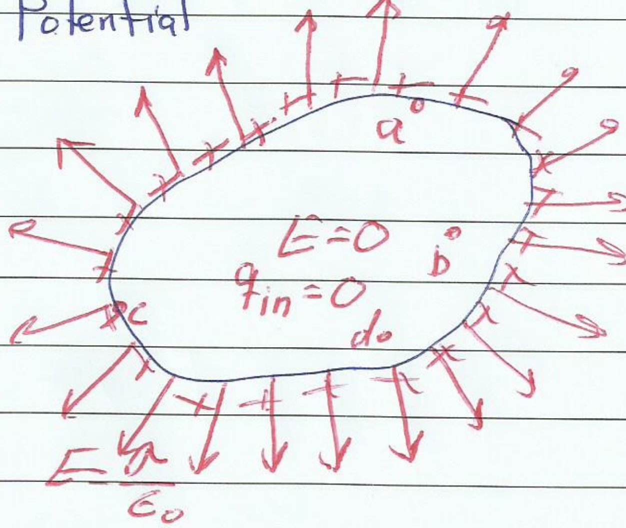
$$= -2.18\times10^{-18}\,J$$

Potential of a Charged Isolated Conductor.

Charged Isolated Conductor:

1) The charge sit at the outer surface
2) Charge inside the Conductor $= 0$
3) E inside the Conductor $= 0$
4) E_{near} the outer surface $= \frac{\sigma}{\epsilon_0}$
5) All its points have the same Potential

$V_a = V_b = V_c = V_d$



Charged Isolated Conducting sphere.

Conducting sphere
- → radius $= R$
- → charge $= +Q$
- → $\sigma = \frac{+Q}{4\pi R^2}$ [All extra charge Q on the surface
- → $\rho = 0$ [No charge inside charged Conducting sphere
- → $E_s = \frac{Q}{4\pi\epsilon_0 R^2}$ or $E_s = \frac{\sigma}{\epsilon_0}$
- → $E_{insid} = 0$, $r < R$
- → $E_{outside} = \frac{Q}{4\pi\epsilon_0 r}$, $r \geq R$
- → $V_s = \frac{+Q}{4\pi\epsilon_0 R}$
- → $V_{inside} = V_s = \frac{+Q}{4\pi\epsilon_0 R}$, $r \leq R$
- → $V_{outside} = \frac{+Q}{4\pi\epsilon_0 r}$, $r \geq R$